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A study of meson-meson potential in the chiral quark model

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Abstract An effective potential in a meson-meson system is discussed based on the SU(3) chiral constituent quark model, and the analytic form of the potential is explicitly given. In addition, the effective potential is employed to study the bound state problem of $\omega\phi$, which is related to the new resonance of $f_0(1810)$ observed in BESII very recently.

Key words Cluster model, SU(3) chiral quark model, molecule states, meson-meson effective potential

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1 Introduction

Recently, many new hadronic states have been discovered, such as light new resonances of X(1835) and $f_0(1810)$ observed by BESII, and the new charmonium or charmonium-like states of X(3872), Y(3940), $Z^+(4430)$ et al. measured in Belle and Babar [1–4]. Since it is difficult to accommodate those new resonances in the conventional quark model, and many of them locate just below the threshold of two mesons, some interpretations, like the molecular picture and tetraquark states [5, 6], which are different from the conventional quark-antiquark constituent quark model, have been proposed to understand their structures.

Since most of the interpretations in the literature are based on the effective Lagrangians in the hadronic level, a more sophisticated understanding of the new resonances in the quark model is required. We know that the SU(3) chiral constituent quark model is one of the most successful quark models which can well reproduce the nucleon-nucleon, nucleon-hyperon interactions and baryon spectroscopy simultaneously [7, 9]. The model considers the one-boson exchange including the scalar and pseudoscalar mesons, and its Lagrangian is constrained by the chiral symmetry. In the most calculations based on the SU(3) chiral constituent quark model for the baryon-baryon interactions [7, 9] and even for the new resonances [10, 11], the RGM or GCM methods are often employed, where the total system is regarded to be a composite system with two clusters and the obtained effective potential is expressed in terms of generator-coordinates. Here we derive a new effective potential between the two clusters in another way with the chiral quark model. Namely, we try to express the effective potential directly in terms of the relative coordinate between the two clusters other than the generator-coordinates. It is expected that the newly obtained potential is a more realistic one. Then, we apply this effective potential to study the bound state problem of a $\omega\phi$ system, which is closely related to the new resonance of $f_0(1810)$ recently observed in the BESII experiments [2].

This paper is organized as follows. In section 2, the analytical effective potential for a meson-meson system is derived based on the SU(3) chiral constituent quark model with one-boson exchange. The numerical result of the effective potential for the $\omega\phi$ system is given in section 3. A discussion of the bound state problem of the $\omega\phi$ system with this potential is given in the last section.

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2 Analytical effective potential for a meson-meson system

For a system with two mesons, in cluster model, we only need consider the interaction between different clusters, that is, two different mesons. As a consequence, the sum of that kind of contributions between different clusters gives our total effective potential (see Fig.1 for an illustration).

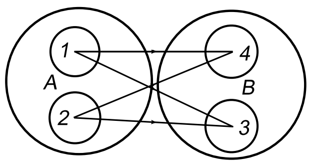

Fig. 1. Interactions between clusters

The total Hamiltonian of such a meson-meson system is

$$H = T + V. \tag{1}$$

Here, we have neglected the interaction potentials between two quarks within each cluster. T is the kinetic energy operator:

$$T = \sum_i T_i - T_{cm} \tag{2}$$

and V is the potential operator which indicates the interactions between the two mesons:

$$\begin{aligned} V &= \sum_{i\in A, j\in B} V_{ij}, & (3)\\ V_{ij} &= V^{OGE}(r_{ij}) + V^{conf}(r_{ij}) + V^{ch}(r_{ij}), & (4)\end{aligned}$$

while $V^{OGE}(r_{ij}), V^{conf}(r_{ij})$ and $V^{ch}(r_{ij})$ are respectively one-gluon-exchange potential, confinement potential and one-meson-exchange potential between i-th quark in cluster A and j-th quark in cluster Bwith $i = (1,2)$ and $j = (3,4)$. The forms of one-gluon-exchange potential and confinement potential have been shown in Ref. [12].

The chiral Lagrangian of the quark-quark interaction under the SU(3) chiral quark model is

$$\mathcal{L}_I^{ch} = -g_{ch} F(\boldsymbol{q}^2)\overline{\psi}(\sum_{a=0}^{8}\lambda_a\sigma_a + i\gamma_5\sum_{a=0}^{8}\lambda_a\pi_a)\psi. \tag{5}$$

Here, λ^a indicates the Gellman flavor matrix, σ^a indicates the scalar masons, π^a stands for pseudoscalar masons, and $F(\boldsymbol{q}^2)$ expresses the form factor of the chiral field with the form of

$$F(\boldsymbol{q}^2) = (\frac{\Lambda^2}{\Lambda^2+\boldsymbol{q}^2})^{1/2}. \tag{6}$$

In non-relativistic limit, we get the quark-quark interaction in momentum space. After the Fourier transformation, we reach the potential in coordinate space. From Ref. [9], we know the central part of the potential with the scalar meson exchanges is

$$V_{cen}^{\sigma_a}(r_{ij}) = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda^2 m_a}{\Lambda^2 - m_a^2} Y_1(r_{ij})\lambda_i^a\lambda_j^a, \tag{7}$$

and the one with the pseudoscalar meson exchange is

$$\begin{aligned} V_{cen}^{\pi_a}(r_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{\Lambda^2 m_a^3}{\Lambda^2 - m_a^2}\frac{1}{m_i m_j}\\ &\quad\times\frac{1}{12}Y_3(r_{ij})(\sigma_i\cdot\sigma_j)\lambda_i^a\lambda_j^a, \qquad (8)\end{aligned}$$

where m_i and m_j denote the masses of i-th quark in cluster A and j-th quark in cluster B. $\lambda^a (a=0,...,8)$ in eqs. (7,8) indicate flavor matrices correspondence to scalar mesons σ^a or pseudoscalar mesons π^a. In the two equations we have

$$\begin{aligned} Y_1(r_{ij}) &= Y(m_a r_{ij}) - \frac{\Lambda}{m_a} Y(\Lambda r_{ij}), & (9)\\ Y_3(r_{ij}) &= Y(m_a r_{ij}) - (\frac{\Lambda}{m_a})^3 Y(\Lambda r_{ij}), & (10)\end{aligned}$$

and the Yukawa function is

$$Y(x) = \frac{1}{x}e^{-x}. \tag{11}$$

Here, m_a denotes the meson mass in scalar or pseudoscalar nonets. Now the chiral potential

$$V^{ch}(r_{ij}) = \sum_a V^{\sigma_a}(r_{ij}) + \sum_a V^{\pi_a}(r_{ij}). \tag{12}$$

The effective potentials in eqs.(7,8) have three parts: orbital, associated with r; spin, associated with $\sigma_i\cdot\sigma_j$ which only appears in the pseudoscalar meson case in eq.(8); flavor, associated with $\lambda_i^a\lambda_j^a$. Except the orbital part, the other two parts are easy to deal with. As a result, we focus our attention on the orbital one in this paper.

Defining the Jacobi coordinates as follows

$$\left\{\begin{array}{lcl} \overrightarrow{\xi_1} &=& \overrightarrow{r_1} - \overrightarrow{r_2}\\ \overrightarrow{R_1} &=& \frac{m_1\overrightarrow{r_1}+m_2\overrightarrow{r_2}}{m_1+m_2}\\ \overrightarrow{\xi_2} &=& \overrightarrow{r_3} - \overrightarrow{r_4}\\ \overrightarrow{R_2} &=& \frac{m_3\overrightarrow{r_3}+m_4\overrightarrow{r_4}}{m_3+m_4}, \end{array}\right. \tag{13}$$

where $\overrightarrow{\xi_1}$ and $\overrightarrow{\xi_2}$ are respectively the relative coordinates within the two clusters, and $\overrightarrow{R_1}$ and $\overrightarrow{R_2}$ the center-of-mass coordinates, the single particle coordinates can then be re-written in terms of the new

Jacobi corrdinates as

$$\left\{\begin{array}{rcl}\overrightarrow{r_1} &=& \overrightarrow{R_1}+\frac{m_2}{m_1+m_2}\overrightarrow{\xi_1}\\ \overrightarrow{r_2} &=& \overrightarrow{R_1}-\frac{m_1}{m_1+m_2}\overrightarrow{\xi_1}\\ \overrightarrow{r_3} &=& \overrightarrow{R_2}+\frac{m_4}{m_3+m_4}\overrightarrow{\xi_2}\\ \overrightarrow{r_4} &=& \overrightarrow{R_2}-\frac{m_3}{m_3+m_4}\overrightarrow{\xi_2}.\end{array}\right. \tag{14}$$

Moreover $\overrightarrow{r_{13}}$ can be written as

$$\overrightarrow{r_{13}}=\overrightarrow{\xi}+\frac{m_2}{m_1+m_2}\overrightarrow{\xi_1}-\frac{m_4}{m_3+m_4}\overrightarrow{\xi_2} \tag{15}$$

with $\overrightarrow{\xi}=\overrightarrow{R_1}-\overrightarrow{R_2}$ being the relative coordinates between the two clusters.

Usually, we take a Guassian-like wavefunction for the orbital wave-function of each quark in the cluster for simplity. It is

$$\varphi(\mathbf{r}) \;=\; (\frac{m_q\omega}{\pi})^{3/4}e^{-\frac{m_q\omega}{2}\mathbf{r}^2}, \tag{16}$$

where $\mathbf{r}$ and m_q stands for the coordinate and the mass of each quark respectively, and the parameter ω is chosen as $0.5GeV\approx 2.522fm^{-1}$ traditionally. The normalized orbital wavefunction of cluster A is

$$\varphi_{A_o}=(\frac{m_1\omega}{\pi})^{3/4}(\frac{m_2\omega}{\pi})^{3/4}e^{-\frac{\omega}{2}(m_1r_1^2+m_2r_2^2)}. \tag{17}$$

It is also true for cluster B. Then the total orbital wavefunction of cluster A and cluster B is

$$|\varphi_{A_o}\varphi_{B_o}\rangle \;=\; (\frac{m_1\omega}{\pi})^{3/4}(\frac{m_2\omega}{\pi})^{3/4}(\frac{m_3\omega}{\pi})^{3/4}(\frac{m_4\omega}{\pi})^{3/4}e^{-\frac{\omega}{2}(m_1r_1^2+m_2r_2^2+m_3r_3^2+m_4r_4^2)}. \tag{18}$$

The wavefunction (18) of the system can be written in terms of the newly defined Jacobi corrdinates in eq.(13) as

$$|\varphi_{A_o}\varphi_{B_o}\rangle \;=\; (\frac{\mu_{12}\omega}{\pi})^{3/4}(\frac{\mu_{34}\omega}{\pi})^{3/4}(\frac{M\omega}{\pi})^{3/4}(\frac{\mu_{12,34}\omega}{\pi})^{3/4}e^{-\frac{\omega}{2}(\mu_{12}\xi_1^2+\mu_{12}\xi_2^2+MR_c^2+\mu_{12,34}\xi^2)}, \tag{19}$$

where

$$\begin{array}{rcl}\mu_{12} &=& \dfrac{m_1m_2}{m_1+m_2}, \qquad (20)\\ \mu_{34} &=& \dfrac{m_3m_4}{m_3+m_4},\\ M &=& m_1+m_2+m_3+m_4,\\ \mu_{12,34} &=& \dfrac{(m_1+m_2)(m_3+m_4)}{m_1+m_2+m_3+m_4},\\ \overrightarrow{\xi} &=& \overrightarrow{R_1}-\overrightarrow{R_2},\\ \overrightarrow{R_c} &=& \dfrac{m_1\overrightarrow{r_1}+m_2\overrightarrow{r_2}+m_3\overrightarrow{r_3}+m_4\overrightarrow{r_4}}{m_1+m_2+m_3+m_4}.\end{array}$$

The total effective potential should only have relevance to the relative coordinates between the two clusters, that is:

$$V(\xi) \;=\; \sum_{ij}V_{ij}(\xi), \tag{21}$$

$$V_{ij}(\overrightarrow{\xi}) \;=\; V_{ij}^{OGE}(\xi)+V_{ij}^{conf}(\xi)+V_{ij}^{ch}(\xi), \tag{22}$$

where $V_{ij}(\xi)$ is the effective interaction between the i-th quark in cluster A and j-th quark in the cluster B. It is

$$V_{ij}(\xi)=\frac{\langle\varphi_{Ao}\varphi_{Bo}|V(r_{ij})\mathcal{A}|\varphi_{Ao}\varphi_{Bo}\rangle}{\langle\varphi_{Ao}\varphi_{Bo}|\mathcal{A}|\varphi_{Ao}\varphi_{Bo}\rangle}, \tag{23}$$

In eq.(23), $\mathcal{A}$ is the anti-symmetrization operator:

$$\mathcal{A} \;=\; (1-P_{13})(1-P_{24}). \tag{24}$$

There are four terms in eq.(23), and here we take direct term as an example for our manipulation which has no relation with exchange operator P_{ij}.

We see in eqs.(7,8) that all the meson exchange potentials are the algebraic sum of two Yukawa potentials with different parameters and coefficients, so we just calculate the effective potential with simple Yukawa form

$$\mathcal{V}(r_{ij})=\frac{e^{-mr_{ij}}}{r_{ij}}, \tag{25}$$

and then

$$\mathcal{V}_{ij}(\xi)=\frac{\langle\varphi_{Ao}\varphi_{Bo}|\frac{e^{-mr_{ij}}}{r_{ij}}\mathcal{A}|\varphi_{Ao}\varphi_{Bo}\rangle}{\langle\varphi_{Ao}\varphi_{Bo}|\mathcal{A}|\varphi_{Ao}\varphi_{Bo}\rangle}. \tag{26}$$

In the case of $i,j=1,3$ we have

$$\begin{aligned}\mathcal{V}_{13}(\xi) &= \int d\overrightarrow{\xi_1}\int d\overrightarrow{\xi_2}\frac{e^{-mr_{13}}}{r_{13}}(\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}e^{-\omega(\mu_{12}\xi_1^2+\mu_{12}\xi_2^2)}\\ &= \int d\overrightarrow{\xi_1}\int d\overrightarrow{\xi_2}\frac{e^{-m|\xi+\frac{m_2}{m_1+m_2}\overrightarrow{\xi_1}-\frac{m_4}{m_3+m_4}\overrightarrow{\xi_2}|}}{|\xi+\frac{m_2}{m_1+m_2}\overrightarrow{\xi_1}-\frac{m_4}{m_3+m_4}\overrightarrow{\xi_2}|}(\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}e^{-\omega(\mu_{12}\xi_1^2+\mu_{12}\xi_2^2)}.\end{aligned} \tag{27}$$

After some manipulations(see Appendix), and taking

$$\beta = \frac{\mu_{12}\mu_{34}\omega}{\mu_{12}(\frac{m_4}{m_3+m_4})^2+\mu_{34}(\frac{m_2}{m_1+m_2})^2}, \qquad (28)$$

we finally reach

$$\mathcal{V}_{13}(\xi) = \frac{1}{2\xi}e^{\frac{m^2}{4\beta}}\{e^{-m\xi}\{1-erf[-\sqrt{\beta}(\xi-\frac{m}{2\beta})]\}-e^{m\xi}\{1-erf[\sqrt{\beta}(\xi+\frac{m}{2\beta})]\}\}. \qquad (29)$$

Similarly, one can easily deduce the other three $\mathcal{V}_{ij}$, while the only difference is the value and the form of β in eq.(28).

Applying this result to eqs.(7-10), we could get the orbital part of the meson-exchange potential in direct term in eq.(23). Applying this method, we could get the one-gluon-exchange potential and confinement potential between the two clusters. The sum of meson-exchange potential, one-gluon-exchange potential and confinement potential gives the contribution of direct term in eq.(23). The process to handle the exchange terms in eq.(23) which are associated with P_{ij} resembles what has been done to the direct term. Other parts of this potential, like the spin and flavor ones could be easily obtained (see Ref. [12]).

3 The numerical result of the effective potential for the $\omega\phi$ system

Here, we will employ our formalism to study the bound state problem of the $\omega\phi$ system. The flavor part of the wavefunction of $\omega\phi$ system is

$$|\varphi_{\omega_f}\varphi_{\phi_f}\rangle = |\frac{1}{\sqrt{2}}(u\overline{u}+d\overline{d})(s,\overline{s})\rangle. \qquad (30)$$

Because there isn't same flavor quarks in the two different color singlet clusters, the one-gluon-exchange potential and confinement potential don't exist in $\omega\phi$ system. Then, the realistic effective potential is the total contribution of meson-exchange potential:

$$V(\xi) = \sum_{ij}\frac{\langle\varphi_\omega\varphi_\phi|V^{ch}(r_{ij})\mathcal{A}|\varphi_\omega\varphi_\phi\rangle}{\langle\varphi_\omega\varphi_\phi|\mathcal{A}|\varphi_\omega\varphi_\phi\rangle}. \qquad (31)$$

Considering the flavor part of $V^{ch}(r_{ij})$ in eqs.(7,8,12), we know that only certain kinds of meson-exchange can exist in eq.(31). From PDG [13], we know: m_K=494MeV, m_η=548MeV, $m_{\eta'}$=958MeV, m_ω=782MeV and m_ϕ=1020MeV. To get a reliable result, we use the following parameters with which the spectroscopy of baryons can be well fitted [7, 8]: the scalar meson masses as m_σ=595MeV, m_ϵ=980MeV, m_κ=980MeV, the cutoff mass as Λ=1100MeV, the quark masses as m_u=313MeV and m_s=470MeV, ω=2.522fm^{-1} and the quark and chiral field coupling constant g_{ch}= 2.621. After elaborate investigation, we know that in direct term of eq.(31), there are four pairs of σ, ϵ, η and η' exchange respectively as shown in Figs.2 with σ shown in the short dot line, ϵ the dash line, η the dot line and η' the dash dot line. In exchange terms of eq.(31), there are two pairs of κ and K exchange respectively as shown in Figs.3 with κ shown in the short dash line, K the dash dot dot line.

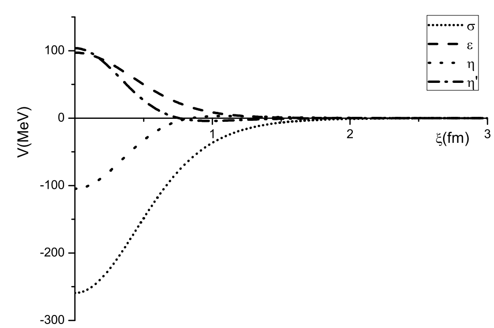

Fig. 2. Direct term's contribution to the total potential

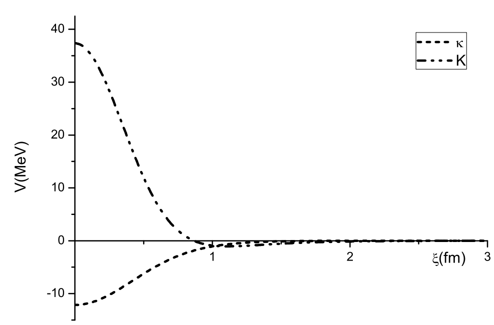

Fig. 3. Exchange term's contribution to the total potential

From Figs.2-3, we see the contributions of σ exchange, η exchange and κ exchange are attractive, while the contributions of ϵ exchange, η' exchange and K exchange are repulsive. These features of the

effective chiral meson exchange potentials are consistent with the results of RGM calculation [11].

Now, we can easily determine the total potential as the sum of the contribution as shown in Figs.2-3. It is shown in Fig.4 with the solid line in comparison with the different part contributions:

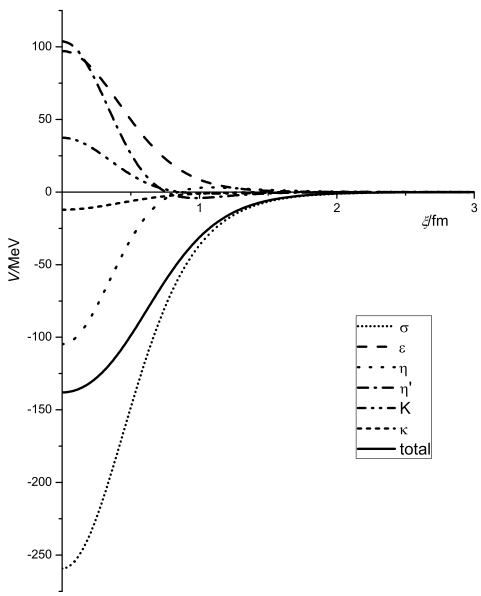

Fig. 4. The total potential and all the different parts' contribution

Finally, we will trace out whether an $\omega\phi$ quasi-bound state can exist. Since we have deduced the realistic effective potential which is expressed in terms of relative coordinate between ω and ϕ and chosen the mass of σ as 595MeV, we can solve the Schrödinger Equation, and work out the existance of eigenfunction and eigenvalue. For this purpose we use the computer program developed by Lucha and Schöberl [14]. Unfortunately, we find that there isn't any eigenfunction or eigenvalue. Then we can draw a conclusion that ω and ϕ can't form a steady system in our approach, which confirms to the conclusion of Ref. [11] We also notice that if the mass of σ is readjusted to 520MeV, this system would have one eigenstate with an eigenvalue of 0.26MeV.

4 Summary

The main purpose of this paper is to deduce an analytical form of the interaction potential between different clusters in SU(3) chiral quark model. Compared with the generator-coordinate method in solving this problem, our method is more realistic. More importantly, the short-range behavior of the two clusters could be clearly worked out. This method could be generalized to a more practical form of the potential which would include the tensor interaction terms and spin-orbital coupling terms. It could be also applied to other hadronic molecules and pentaquark systems.

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Appendix

For eq.(27), to further simplify our derivation, we make another coordinates transformations:

$$\left\{ \begin{array}{lcl} \overrightarrow{X} &=& \overrightarrow{\xi} + \frac{m_2}{m_1+m_2}\overrightarrow{\xi_1} - \frac{m_4}{m_3+m_4}\overrightarrow{\xi_2} \\ \overrightarrow{Y} &=& \frac{m_3+m_4}{2m_4}\overrightarrow{\xi_1} + \frac{m_1+m_2}{2m_2}\overrightarrow{\xi_2}. \end{array} \right. \tag{32}$$

The inverse transformations are

$$\left\{ \begin{array}{lcl} \overrightarrow{\xi_1} &=& \frac{m_4}{m_3+m_4}\overrightarrow{Y} + \frac{m_1+m_2}{2m_2}(\overrightarrow{X} - \overrightarrow{\xi}) \\ \overrightarrow{\xi_2} &=& \frac{m_2}{m_1+m_2}\overrightarrow{Y} - \frac{m_3+m_4}{2m_4}(\overrightarrow{X} - \overrightarrow{\xi}), \end{array} \right. \tag{33}$$

Then, $\mathcal{V}_{13}$ is

$$
\begin{aligned}
\mathcal{V}_{13}(\xi) &= (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{-\mu_{12}\omega[\frac{m_4}{m_3+m_4}\overrightarrow{Y}+\frac{m_1+m_2}{2m_2}(\overrightarrow{X}-\overrightarrow{\xi})]^2} \\
&\quad \times e^{-\mu_{34}\omega[\frac{m_2}{m_1+m_2}\overrightarrow{Y}-\frac{m_3+m_4}{2m_4}(\overrightarrow{X}-\overrightarrow{\xi})]^2} \\
&= (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{-\mu_{12}\omega[(\frac{m_4}{m_3+m_4})^2Y^2+(\frac{m_1+m_2}{2m_2})^2(\overrightarrow{X}-\overrightarrow{\xi})^2]} \\
&\quad \times e^{-\mu_{34}\omega[\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}\overrightarrow{Y}\cdot(\overrightarrow{X}-\overrightarrow{\xi})]}e^{-\mu_{12}\omega[(\frac{m_2}{m_1+m_2})^2Y^2+(\frac{m_3+m_4}{2m_4})^2(\overrightarrow{X}-\overrightarrow{\xi})^2]}e^{\mu_{34}\omega[\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}\overrightarrow{Y}\cdot(\overrightarrow{X}-\overrightarrow{\xi})]} \\
&= (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{-\mu_{12}\omega[(\frac{m_4}{m_3+m_4})^2Y^2+(\frac{m_1+m_2}{2m_2})^2(\overrightarrow{X}-\overrightarrow{\xi})^2]} \\
&\quad \times e^{-\mu_{12}\omega[\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}\overrightarrow{Y}\cdot(\overrightarrow{X}-\overrightarrow{\xi})]}e^{-\mu_{34}\omega[(\frac{m_2}{m_1+m_2})^2Y^2+(\frac{m_3+m_4}{2m_4})^2(\overrightarrow{X}-\overrightarrow{\xi})^2]}e^{\mu_{34}\omega[\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}\overrightarrow{Y}\cdot(\overrightarrow{X}-\overrightarrow{\xi})]} \\
&= (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{-[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]Y^2} \\
&\quad \times e^{-[\mu_{12}\omega(\frac{m_1+m_2}{2m_2})^2+\mu_{34}\omega(\frac{m_3+m_4}{2m_4})^2](\overrightarrow{X}-\overrightarrow{\xi})^2}e^{[-\mu_{12}\omega\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}+\mu_{34}\omega\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}]\overrightarrow{Y}\cdot(\overrightarrow{X}-\overrightarrow{\xi})}. \qquad (34)
\end{aligned}
$$

According to

$$aY^2+bX^2+cX\cdot Y=a(Y+\frac{c}{2a}X)^2+(b-\frac{c^2}{4a})X^2, \quad (35)$$

we have

$$\mathcal{V}_{13}(\xi) = (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{M}e^{N}, \qquad (36)$$

where

$$
\begin{aligned}
M &= -[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]\left\{Y+\frac{\mu_{12}\omega\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}-\mu_{34}\omega\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}}{2[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]}(\overrightarrow{X}-\overrightarrow{\xi})\right\}^2 \\
&= -[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]Y'^2, \\
N &= -\left\{\mu_{12}\omega(\frac{m_1+m_2}{2m_2})^2+\mu_{34}\omega(\frac{m_3+m_4}{2m_4})^2-\frac{[\mu_{12}\omega\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}-\mu_{34}\omega\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}]^2}{4[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]}\right\}(\overrightarrow{X}-\overrightarrow{\xi})^2 \\
&= -\frac{\mu_{12}\mu_{34}\omega}{\mu_{12}(\frac{m_4}{m_3+m_4})^2+\mu_{34}(\frac{m_2}{m_1+m_2})^2}(\overrightarrow{X}-\overrightarrow{\xi})^2. \qquad (37)
\end{aligned}
$$

Furthermore, after making another coordinate transformation to Y as

$$
\begin{aligned}
Y' &= \frac{\mu_{12}\omega\frac{m_4(m_1+m_2)}{m_2(m_3+m_4)}-\mu_{34}\omega\frac{m_2(m_3+m_4)}{m_4(m_1+m_2)}}{2[\mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2]}(\overrightarrow{X}-\overrightarrow{\xi}) \\
&\quad +Y \qquad (38)
\end{aligned}
$$

and taking

$$
\begin{aligned}
\alpha &= \mu_{12}\omega(\frac{m_4}{m_3+m_4})^2+\mu_{34}\omega(\frac{m_2}{m_1+m_2})^2, \qquad (39) \\
\beta &= \frac{\mu_{12}\mu_{34}\omega}{\mu_{12}(\frac{m_4}{m_3+m_4})^2+\mu_{34}(\frac{m_2}{m_1+m_2})^2},
\end{aligned}
$$

we get

$$\mathcal{V}_{13}(\xi) = (\frac{\mu_{12}\omega}{\pi})^{3/2}(\frac{\mu_{34}\omega}{\pi})^{3/2}\int d\overrightarrow{X}\int d\overrightarrow{Y}\frac{e^{-mX}}{X}e^{-\alpha Y^2}e^{-\beta(\overrightarrow{X}-\overrightarrow{\xi})^2} \qquad (40)$$

Notice that

$$
\begin{aligned}
\int d\overrightarrow{X}\frac{1}{X}e^{-mX-\beta(\overrightarrow{X}-\overrightarrow{\xi})^2} &= e^{-\beta\xi^2}\int d\overrightarrow{X}\frac{1}{X}e^{-mX-\beta X^2+2\overrightarrow{X}\cdot\overrightarrow{\xi}} \\
&= \frac{\pi\sqrt{\pi}}{2\beta\sqrt{\beta}\xi}e^{\frac{m^2}{4\beta}}\{e^{-m\xi}\{1-erf[-\sqrt{\beta}(\xi-\frac{m}{2\beta})]\}-e^{m\xi}\{1-erf[\sqrt{\beta}(\xi+\frac{m}{2\beta})]\}\} \qquad (41) \\
\int d\overrightarrow{Y}e^{-\alpha Y^2} &= \frac{\pi\sqrt{\pi}}{\alpha\sqrt{\alpha}} \qquad (42)
\end{aligned}
$$

We finally get

$$\mathcal{V}_{13}(\xi) = \frac{1}{2\xi}e^{\frac{m^2}{4\beta}}\{e^{-m\xi}\{1-erf[-\sqrt{\beta}(\xi-\frac{m}{2\beta})]\}-e^{m\xi}\{1-erf[\sqrt{\beta}(\xi+\frac{m}{2\beta})]\}\}, \tag{43}$$

and this is what we want in eq.(29).

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